

**IMPLEMENTATION OF UNUSED PRODUCTION  
FACTORS IN AGRICULTURE BY MEANS  
OF DYNAMIC OPTIMIZATION MODELS  
WITH RANDOM CONSTRAINTS**

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**Abstract.** The farms of Western Pomerania province possess a large surplus of manpower. The dynamic optimization models with random constraints served the investigation of the possibilities of implementation of the unused man-hours. Those models regarded four successive years 2003-2006. The solution proceeded in two steps. The first step let us construct the assumption of the surplus or the deficiency of production factors. In the next step additional variables regarding the lease of arable grounds were introduced while the unused man-hours were implemented with various probability. The optimal solutions indicated the area of particular crops, the quantity of livestock and the farm income dependent on the use of the existing employment. This study aims at the presentation of the possibility of implementation of unused man-hours in farms dealing solely with the crop production and also the production of crop and livestock.

**Key words:** dynamic optimization model, stochastic programming, farm income, structure of agricultural production

**INTRODUCTION**

As a result of changes and structural transformations in Poland the problem of unemployment emerged in rural areas. The problem did not omit Western Pomerania province, once dominated by collective farming (state and cooperative farms), abolished in the 90s of the previous century. Worth noticing is the fact that the farmers having farms of over 2 ha arable land are not considered unemployed. This constitutes so-called

hidden unemployment. Western Pomerania province is characterised [Program... 2005] by the highest, but one, percentage of unemployment – 27.4% right after Ermland-Masuria province. Whereas the employment on 100ha of arable land equals 7.1 people and is the lowest in the country. Despite that the farms of Western Pomerania province have a surplus of manpower. This study aims at the indication of the possibilities of the implementation of unused man-hours in the farms dealing with the production of crops solely, as well as the crops and the livestock. The study used the dynamic optimization models with random constraints. They allowed to assume such a production structure, provided a various probability of accomplishment of production factors, which gives the highest farm income. One of the first studies presenting solutions with constraints in the form of probabilities was that by Tintner and Sengupta [1972]. The theoretical bases of the construction of the models with random constraints was shown by Krawiec [1991]. The optimization of agricultural production was the subject of Wąs [2005], Zieliński [2009] among others, yet those were mainly the linear programming models. The world literature comprises a number of examples of the implementation of the stochastic programming in agriculture. For instance, Paudyal and Gupta [2003] presented the project of the irrigation of the agricultural acreage by means of the non-linear model, considering a random quantity of rainfall.

## RESEARCH METHOD

The research method comprises the dynamic optimization models with random constraints [Grabowski 1980]. Let us consider the model:

$$Ax \leq (\geq) b \text{ balance constraints} \quad (1)$$

$$x \geq 0 \text{ boundary constraint} \quad (2)$$

$$Fc = c^T x \rightarrow \max \text{ objective function} \quad (3)$$

where:

$A$  – technical-economic parameters,

$x$  – decisive variables,

$c$  – objective function coefficients,

$b$  – random vector of evident probability distribution.

Assuming that the constraints vector  $b$  is a random variable of jumping type (if the  $b$  vector is a continuous random variable, it should be subject to discreditation). Let  $b_i$  for  $i = 1, 2, \dots, m$  denote the accomplishment of  $b$  random vector in the constraints denoted by the dependency (1). Considering the  $i$ -th constraint we will meet one of the three situations:

$$a_i x_i = b_i; a_i x_i > b_i; a_i x_i < b_i$$

The  $i$ -th constraint is exactly fulfilled in the first case, in the second one there is a deficiency of resources  $a_i x_i = b_i + y_i$  i.e.  $a_i x_i - y_i = b_i$ , whereas in the third one their surplus  $a_i x_i = b_i - y_i$  i.e.  $a_i x_i + y_i = b_i$ , where  $y_i$  is a variable of  $i$ -th resource of the deficiency and/or the surplus. Both the surplus and the deficiency in the accomplishment of the constraints is encumbered with  $k_i$  costs.

Such a task is solved in two-steps. The first step assumes conventionally that the random variables in the constraints vector assume some determined values (eg. at the level of their expected values). Further, by means of the linear programming methods the optimal solution is calculated.

The second step comprises the observation of the accomplishment of  $b$  random vector and the evaluation of the divergence (deficiency and/or surplus). The divergence is calculated by the vector

$$By = b - Ax^*$$

where:

$x^*$  – decisive variables from the optimal solution in the first step,

$B$  – the singular matrix of  $m$  degree.

The problem of this step can be written as:

$$By = b - Ax^* \quad (4)$$

$$y \geq 0 \quad (5)$$

$$k^T y \rightarrow \min \quad (6)$$

The optimal solution of the entire problem is obtained by solving the model:

$$Ax + By = b \quad (7)$$

$$x \geq 0, y \geq 0 \quad (8)$$

$$Fc = c^T x + (\min k^T y) \rightarrow \max \quad (9)$$

The result of the solution is the vector of decisive variables  $x^*$  and the expected deviations from the accomplishment  $y^*$ , provided the costs of the deviations from the accomplishment of  $b$  vector are possibly the lowest.

The models constructed by means of the assumptions (7)-(9) for each investigated year are connected with each other by means of the mutual (binding) constraints resulting in the dynamic optimization model with random constraints. The binding constraints fulfill the principle of Bellman [Bellman and Dreyfus 1967] recurrence equations.

## DYNAMIC OPTIMIZATION MODEL OF A FARM WITH RANDOM CONSTRAINTS

The research material constituted the database of Statistical Office, Agency of Restructuring and Modernization of Agriculture and Regional Agricultural Advisory Center on Western Pomerania province in the years 2003-2006. The information regarded (mean qualities in parentheses):

- the area of arable grounds (13.14 ha) and permanent greens (3.23 ha),
- the structure of stands (cereals – 59%, potatoes – 2%, beats – 2%, rape – 19%, other crops – 11%, fallows – 15%),
- the livestock *per capita* (cattle 4 adult specimens, swine 3-4 adult specimens),
- the employment (3 people),
- the singular input, the price and the efficiency.

On the basis of the data four classical linear programming models were built. They described an average farm of Western Pomerania dealing with sole crop production in

the successive years. Those models were connected with each other by the balance (mutual) constraints regarding the crop rotation [Zaród 2008]. The dynamic model of four joined blocks (each year constitutes a separate block) consisted of 44 decisive variables and 47 constraints. The objective function of that model was the brut farm income constituting the difference between the production value and the direct costs (sowing material, foil, taxes, insurance) without the price of the farmer's own work. The calculations of the singular production values and the costs of particular agricultural activities were based on the elaborations of Western Pomerania Branch of Agricultural Advisory Center [Kalkulacje... 2003-2006]. The farm income for the years 2004 and 2005 was increased by the direct and supplementary subsidies while for 2006 additionally by the sugar charges.

The solution of this model ran in two-steps. The first step consisted in the solution of the dynamic linear programming model. The solution resulted in the quantity of the unused man-hours in the peak demand of the manpower, besides the area of particular crops and the value of income. In the second step new variables were introduced which regarded the lease of a supplementary area of arable grounds to utilize the existing resources of productive factors. The dimensions of the model were enlarged to 56 decisive variables and 71 balance constraints. The objective function of the added variables was encumbered by the increased costs resulting from the lease charges and the land tax. Moreover, the accomplishment of the constraints regarding the man-hours assumed the values from the interval  $(d_1; d_2)$ , where  $d_1$  – the quantity of man-hours necessary to sustain the production at the level stated by the linear programming model,  $d_2$  – real quantity of man-hours in a farm. The unused man-hours were assumed to be realized with the probability  $p_i = 0.25; 0.50; 0.75; 1$ . The possibilities of increasing the production factors and the achieved farm income in four analysed years jointly in the optimal solutions are shown in Table 1.

Table 1. Area of arable land and income from the optimal solutions  
Tabela 1. Powierzchnia gruntów ornych i dochód z rozwiązań optymalnych

Specification Wyszczególnienie	Probability of realization of unused man-hours Prawdopodobieństwo realizacji niewykorzystanych roboczogodzin				
	p = 0	p = 0.25	p = 0.50	p = 0.75	p = 1
Area of arable land (ha): Powierzchnia gruntów ornych (ha):					
2003	12.79	18.84	25.68	32.32	38.91
2004	12.66	19.38	26.14	32.82	39.29
2005	13.43	19.87	26.44	33.00	39.86
2006	13.68	20.03	26.97	33.90	40.64
Average value (ha) Wartość średnia (ha)	13.14	19.53	26.31	33.01	39.67
Farm income (PLN) Dochód rolniczy (zł)	63 464.34	89 173.27	105 783.69	121 595.57	137 391.47

Source: author's own calculations on MATLAB.  
Źródło: obliczenia własne za pomocą pakietu MATLAB.

Each of the successive solutions increases: the area of arable land by over 6 ha and the farm income by 25 708.93, 16 610.42, 15 811.88, 15 795.90 PLN, respectively. The highest increase of the income causes an increase of the unused man-hours by 25% and a further increase of this production factor causes a comparable increase of profitability. A precise analysis of the income converted to 1 ha arable grounds in four years is presented on the Figure 1.

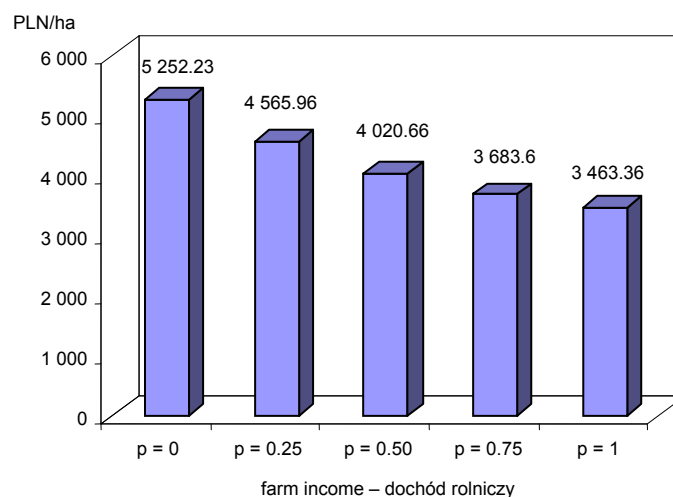


Fig. 1. Singular farm income in the models of varied implementation of unused manpower

Rys. 1. Jednostkowy dochód rolniczy w modelach o różnym zagospodarowaniu niewykorzystanej siły roboczej

A singular farm income decreases with the increase of the farm area. It results mainly from the charges of leased land and the structure of crops. Table 2 presents the area of particular crops from the optimal solution of the linear model ( $p = 0$ ) and two models with random constraints (of incomplete ( $p = 0.5$ ) and complete ( $p = 1$ ) implementation of the unused man-hours) All the solutions induce the principle of the correct crop rotation.

The optimal solution of the dynamic deterministic model contains the fallows. They emerge from the real structure of the land usage in Western Pomerania. They were encumbered by the land tax in 2003, while in the remaining years they were subject of the direct subsidies (they were assumed to be in a good agricultural culture). The solution of the linear model indicated a large number of unused man-hours, even in the periods of increased demand for manpower (crop harvest, potato lifting). The solutions of the model with random constraints allow the implementation of those production factors. A full usage of the existing employment in farms dealing with crop production solely, lets the enlargement of the area of the arable grounds three times. The enlarged area, by means of the lease, should be mainly destined for the cultivation of crops (relatively low costs) and potatoes and/or beats (most profitable production directions).

Table 2. Rotation and area of crops in optimal solutions for  $p = 0, 0.5, 1$   
 Tabela 2. Zmianowanie i powierzchnia upraw w rozwiązaniach optymalnych dla  $p = 0, 0.5, 1$

Year Rok	Solution of model from step I ( $p = 0$ ) – Rozwiązanie modelu z etapu I ( $p = 0$ )									
	Field I (ha) Pole I (ha)	Field II (ha) Pole II (ha)	Field III (ha) Pole III (ha)	Field IV (ha) Pole IV (ha)	Fallows (ha) Odłogi (ha)					
1	2	3	4	5	6					
2003	Potatoes Ziemniaki	0.30	Wheat Pszenica	1.58	Rape Rzepak	1.15	Rye Żyto	2.70	2.43	
	Beets Buraki	0.26	Barley Jęczmień	–	Triticale Pszenżyto	0.32	Other crops Inne uprawy	1.74		
	Oats Owies	2.31								
2004	Wheat Pszenica	2.87	Rape Rzepak	1.27	Rye Żyto	–	Potatoes Ziemniaki	0.68	2.30	
	Barley Jęczmień	–	Triticale Pszenżyto	0.31	Other crops Inne uprawy	1.47	Beets Buraki	–		
							Oats Owies	3.76		
2005	Rape Rzepak	1.21	Rye Żyto	–	Potatoes Ziemniaki	0.3	Wheat Pszenica	3.13	3.07	
	Triticale Pszenżyto	1.66	Other crops Inne uprawy	1.58	Beets Buraki	0.27	Barley Jęczmień	1.31		
					Oats Owies	0.90				
2006	Rye Żyto	2.05	Potatoes Ziemniaki	0.41	Wheat Pszenica	1.47	Rape Rzepak	1.50	3.32	
	Other crops Inne uprawy	0.82	Beets Buraki	0.27	Barley Jęczmień	–	Triticale Pszenżyto	2.94		
			Oats Owies	0.90						
	Solution of model for $p = 0.5$ – Rozwiązanie modelu dla $p = 0.5$									
	Field I (ha) Pole I (ha)	Field II (ha) Pole II (ha)	Field III (ha) Pole III (ha)	Field IV (ha) Pole IV (ha)	Lease (ha) Dzierżawa (ha)					
2003	Potatoes Ziemniaki	0.37	Wheat Pszenica	2.74	Rape Rzepak	2.31	Rye Żyto	3.13	Cereals Zboża	11.23
	Beets Buraki	0.51	Barley Jęczmień	–	Triticale Pszenżyto	–	Other crops Inne uprawy	1.74	Potatoes Ziemniaki	1.66
	Oats Owies	1.99								
2004	Wheat Pszenica	2.87	Rape Rzepak	2.61	Rye Żyto	0.84	Potatoes Ziemniaki	0.39	Cereals Zboża	11.36
	Barley Jęczmień	–	Triticale Pszenżyto	–	Other crops Inne uprawy	1.47	Beets Buraki	–	Potatoes Ziemniaki	2.12
							Oats Owies	4.48		

Table 2 – cont. / Tabela 2 – c.d.

1	2		3		4		5		6	
2005	Rape Rzepak	2.38	Rye Żyto	1.03	Potatoes Ziemniaki	–	Wheat Pszenica	3.13	Cereals Zboża	11.70
	Triticale Pszenżyto	0.49	Other crops Inne uprawy	1.58	Beets Buraki	1.33	Barley Jęczmień	1.74	Beets Buraki	2.08
2006	Rye Żyto	2.05	Potatoes Ziemniaki	–	Wheat Pszenica	2.31	Rape Rzepak	2.97	Cereals Zboża	12.17
	Other crops Inne uprawy	0.82	Beets Buraki	0.54	Barley Jęczmień	–	Triticale Pszenżyto	1.90	Beets Buraki	2.14
	Solution of model for p = 1 Rozwiązanie modelu dla p = 1									
	Field I (ha) Pole I (ha)		Field II (ha) Pole II (ha)		Field III (ha) Pole III (ha)		Field IV (ha) Pole IV (ha)		Lease (ha) Dzierżawa (ha)	
2003	Potatoes Ziemniaki	0.37	Wheat Pszenica	1.93	Rape Rzepak	2.80	Rye Żyto	2.73	Cereals Zboża	22.98
	Beets Buraki	0.78	Barley Jęczmień	–	Triticale Pszenżyto	–	Other crops Inne uprawy	1.74	Potatoes Ziemniaki	3.14
	Oats Owies	2.44								
2004	Wheat Pszenica	2.87	Rape Rzepak	1.80	Rye Żyto	1.33	Potatoes Ziemniaki	0.39	Cereals Zboża	22.01
	Barley Jęczmień	0.72	Triticale Pszenżyto	–	Other crops Inne uprawy	1.47	Beets Buraki	–	Potatoes Ziemniaki	4.62
							Oats Owies	4.08		
2005	Rape Rzepak	3.59	Rye Żyto	0.22	Potatoes Ziemniaki	–	Wheat Pszenica	3.13	Cereals Zboża	25.20
	Triticale Pszenżyto	–	Other crops Inne uprawy	1.58	Beets Buraki	2.80	Barley Jęczmień	1.34	Beets Buraki	2.00
					Oats Owies	–				
2006	Rye Żyto	2.77	Potatoe Ziemniaki	–	Wheat Pszenica	2.80	Rape Rzepak	4.47	Cereals Zboża	23.20
	Other crops Inne uprawy	0.82	Beets Buraki	0.81	Barley Jęczmień	–	Triticale Pszenżyto	–	Beets Buraki	4.78
			Oats Owies	0.99						

Source: author's own calculations on MATLAB.

Źródło: obliczenia własne za pomocą pakietu MATLAB.

### MODIFICATION OF A FARM MODEL WITH RANDOM CONSTRAINTS

Merely 16% farms of Western Pomerania keep the livestock (cattle, swine). This is caused by abandoning the milk and meat production in small farms and the introduction of higher quality demands adapted to EU standards, among other changes in a farm structure. In order to investigate the usage of manpower in such farms, variables regarding the livestock production were added to the constructed dynamic model. The state of the adult stock was assumed at the level of average qualities in the analysed years in the province, whereas the remaining quantity of animals resulted from the closed turnover of the stock. The livestock was fed with the home made fodder excluding the high protein mixtures. The excessive products were subject to sale. The commercial production included also the wheat and the rape. The binding constraints of the model were increased by the turnover of the stock. The entire model contained 100 decisive variables and 122 constraints. The optimal solution of this model by means of the linear programming (step I) indicated that a farm comprises a scarce surplus of manpower in the periods of peak demands and a deficiency of beats and potatoes. In the second step additional variables were introduced which regarded the lease of land and the purchase of animals and a full accomplishment of unused man-hours was assumed. The model enlarged its dimensions to 124 variables and 146 constraints. The optimal solution of the model with random constraints indicates that the purchase of animals was unprofitable. The possibilities of an extension of arable land and the achieved farm income are shown in Table 3.

Table 3. Area of arable grounds and income from optimal solutions  
Tabela 3. Powierzchnia gruntów ornych i dochód z rozwiązań optymalnych

Specification Wyszczególnienie	Probability of realisation of unused man-hours Prawdopodobieństwo realizacji niewykorzystanych roboczogodzin	
	p = 0	p = 1
Area of arable grounds (ha): Powierzchnia gruntów ornych (ha):		
2003	12.79	14.52
2004	12.66	13.92
2005	13.43	14.03
2006	13.68	16.75
Average value (ha) Wartość średnia (ha)	13.14	14.81
Farm income (PLN) Dochód rolniczy (zł)	119 425.90	128 102.23
Income on 1 ha (PLN) Dochód na 1 ha (zł)	9 088.73	8 649.71

Source: author's own calculations on MATLAB.  
Źródło: obliczenia własne za pomocą pakietu MATLAB.



Labor-consuming livestock breeding limited the abilities to lease larger areas of land. The increase of a farm area by about 2 ha causes an increase of the farm income by 8676.33 PLN during four years. That income converted to 1 ha decreases insignificantly with the increase of the area of arable grounds. The entire accomplishment of the unused man-hours decreases a singular farm income by 4.83%. The area of crops from the optimal solutions of the model regarding the crop and livestock production is shown in Table 4.

Table 4. Rotation and area of crops in optimal solutions

Tabela 4. Zmianowanie i powierzchnia upraw w rozwiązaniach optymalnych

Rok Year	Solution of model from step I (p = 0) – Rozwiązanie modelu z etapu I (p = 0)									
	Field I (ha) Pole I (ha)		Field II (ha) Pole II (ha)		Field III (ha) Pole III (ha)		Field IV (ha) Pole IV (ha)		Fallows (ha) Odłogi (ha)	
1	2		3		4		5		6	
2003	Potatoes Ziemniaki	0.26	Wheat Pszenica	1.48	Rape Rzepak	1.15	Rye Żyto	0.02	2.43	
	Beets Buraki	0.38	Barley Jęczmień	–	Triticale Pszenżyto	5.23	Other crops Inne uprawy	1.66		
	Oats Owies	0.18								
2004	Wheat Pszenica	0.82	Rape Rzepak	0.99	Rye Żyto	4.86	Potatoes Ziemniaki	0.38	2.30	
	Barley Jęczmień	–	Triticale Pszenżyto	0.49	Other crops Inne uprawy	1.52	Beets Buraki	0.25		
							Oats Owies	1.05		
2005	Rape Rzepak	0.15	Rye Żyto	–	Potatoes Ziemniaki	0.4	Wheat Pszenica	1.68	3.07	
	Triticale Pszenżyto	0.67	Other crops Inne uprawy	1.48	Beets Buraki	0.27	Barley Jęczmień	–		
					Oats Owies	5.71				
2006	Rye Żyto	–	Potatoes Ziemniaki	0.41	Wheat Pszenica	3.38	Rape Rzepak	1.50	3.32	
	Other crops Inne uprawy	0.82	Beets Buraki	0.28	Barley Jęczmień	3.0	Triticale Pszenżyto	0.18		
			Oats Owies	0.79						
	Solution of model for p = 1 Rozwiązanie modelu dla p = 1									
	Field I (ha) Pole I (ha)		Field II (ha) Pole II (ha)		Field III (ha) Pole III (ha)		Field IV (ha) Pole IV (ha)		Lease (ha) Dzierżawa (ha)	
2003	Potatoes Ziemniaki	0.18	Wheat Pszenica	2.72	Rape Rzepak	1.31	Rye Żyto	–	Cereals Zboża	2.89

Table 4 – cont. / Tabela 4 – cd.

1	2		3		4		5		6	
2004	Beets Buraki	0.29	Barley Jęczmień	–	Triticale Pszenżyto	2.07	Other crops Inne uprawy	1.89	Beets Buraki	0.12
	Oats Owies	3.05								
	Wheat Pszenica	2.87	Rape Rzepak	1.39	Rye Żyto	1.71	Potatoes Ziemniaki	0.23	Cereals Zboża	2.34
	Barley Jęczmień	0.65	Triticale Pszenżyto	1.33	Other crops Inne uprawy	1.67	Beets Buraki	0.28	Beets Buraki	0.07
2005	Rape Rzepak	1.26	Rye Żyto	1.18	Potatoes Ziemniaki	–	Wheat Pszenica	1.89	Cereals Zboża	2.33
	Triticale Pszenżyto	2.26	Other crops Inne uprawy	1.54	Beets Buraki	0.28	Barley Jęczmień	–	Potatoes Ziemniaki	0.14
					Oats Owies	3.1			Beets Buraki	0.05
2006	Rye Żyto	2.51	Potatoes Ziemniaki	–	Wheat Pszenica	3.38	Rape Rzepak	1.84	Cereals Zboża	3.76
	Other crops Inne uprawy	1.1	Beets Buraki	0.34	Barley Jęczmień	–	Triticale Pszenżyto	0.05	Potatoes Ziemniaki	1.23
			Oats Owies	2.38					Beets Buraki	0.25

Source: author's own calculations on MATLAB.

Źródło: obliczenia własne za pomocą pakietu MATLAB.

The augmented area, by the lease and the implementation of fallows, should be mainly used for the cultivation of fodder crops and root crops (potatoes and beets).

The livestock production significantly increased the singular farm income. In the model with the entire accomplishment of the unused man-hours, it was twice as high as in a similar model regarding the sole crop production. The optimal solution indicates that the livestock breeding was profitable in all analysed years and the stock consisted of 4 cows, 3.92 calves (coefficient of calving = 0.98), 3.12 specimens of feeder calves, 0.8 replacement heifer and culled cow (5-year-period of usage of adult specimen). Fractional qualities of particular specimens prove the absence of a given animal in a farm throughout the entire year. The swine breeding was profitable merely in the first two analysed years. In 2003 the herd consisted of 4 sows, 64 piglets, 62 fatteners, 1 replacement gilt and 1 culled sow. In 2004 the state of the herd decreased to 3 sows, 48 piglets and 62 fatteners while in 2005 merely 46 fatteners were left, reclassified from the previous year piglets.

## CONCLUSIONS

1. The models with random constraints show the unused production factors and indicate possibilities of their accomplishment. Their two-step solution causes a considerable increase of the problem dimensions, the solution enables the computer program MATLAB.

2. Full accomplishment of the unused man-hours creates possibilities to lease an additional area of arable grounds. The farms dealing with the crop production can enlarge their area three times. The farms dealing with the crop and livestock production can enlarge their area by 12.7%.

3. The livestock production increases the farm income from 1 ha of arable grounds by 73.04% in the situation of the unused man-hours, and 2.5 times with the full accomplishment of employment.

4. A singular farm income decreases with the increase of the arable grounds area. This results mainly from additional costs imposed on the leased land (lease charge, land tax).

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## ZAGOSPODAROWANIE NIEWYKORZYSTANYCH CZYNNIKÓW PRODUKCJI W ROLNICTWIE ZA POMOCĄ DYNAMICZNYCH MODELI OPTYMALIZACYJNYCH Z LOSOWYMI OGRANICZENIAMI

**Streszczenie.** W gospodarstwach rolnych województwa zachodniopomorskiego istnieją duże rezerwy siły roboczej. Za pomocą dynamicznych modeli optymalizacyjnych z losowymi ograniczeniami badano możliwości zagospodarowania niewykorzystanych roboczogodzin. Modele te dotyczyły czterech kolejnych lat: 2003-2006. Ich rozwiązanie przebiegało dwuetapowo. Pierwszy etap pozwolił ustalić nadmiar lub niedobór czynników

produkcji. W drugim etapie wprowadzono dodatkowe zmienne, dotyczące wydzierżawienia gruntów ornych, a niewykorzystane roboczogodziny realizowano z różnym prawdopodobieństwem. Rozwiązania optymalne wskazywały powierzchnię poszczególnych upraw, liczbę hodowanych zwierząt i wysokość dochodu rolniczego w zależności od stopnia wykorzystania istniejącego zatrudnienia. Celem tej pracy jest pokazanie możliwości zagospodarowania niewykorzystanych roboczogodzin w gospodarstwach rolnych, zajmujących się tylko produkcją roślinną oraz produkcją roślinną i zwierzęcą.

**Słowa kluczowe:** dynamiczny model optymalizacyjny, programowanie stochastyczne, dochód rolniczy, struktura produkcji rolnej

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